Metacognitive Processes in Successful Mathematical Problem Solving

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Abstract

Problem solving has become the 21st century core value in learning mathematics, but improving one's capability in succeeding in it is a continuing quest. In response to this need, this study defines the presence and interrelationships of the metacognitive processes in successful mathematical problem solving (MPS). It seeks to describe the qualities of the metacognitive solutions that led to the right solution, identifies factors that contributed to the emergence of these qualities, and determines the stages of problem solving which require the said metacognitive processes. Guided by a post-positivistic perspective, the study found the covert cognitive phenomenon from 11 key informants. Strauss & Corbin grounded theory approach was utilized to guide and attain the purpose of this study through self-report and interviews, document analysis, and observation. Ethics requirements were addressed. Experts' participation from the fields of mathematics, psychology, and education ensured the validity and reliability of the methodology. Findings resulted to seven metacognitive processes themes. The metacognitive solution qualities are the ten micro-metacognitive processes found to be regulating the first six metacognitive processes influencing the emergence of the three macro-metacognitive stages in successful MPS. The interrelationships among these metacognitive processes describe the emerging theoretical framework grounded within conditions set by this study. Implications and recommendations emphasize integration of new findings in the teaching-learning process, curriculum review and conduct of further studies and local researches.

Keywords: *metacognitive processes, mathematical problem solving*

1. Introduction

Despite almost innumerable studies conducted to improve problem solving skills or ability-- especially in the field of mathematics—the hope to achieve such is still a challenging one for most learners, not only in this country but globally (Divinagracia, 2010; Jitendra & Star, Jon, 2012; Wongwanich & Sujiva, 2014).

A review of related literature and studies on problem solving conducted from early 20th century to early 21st century relatively showed the assumed presence and demand of metacognition as one of the important aspects of successful MPS. In fact, much has been done already in terms of studying about difficulties encountered, knowledge and skills needed in what particular domain or aspect of math problems, strategies helpful in learning MPS, and many other concepts related to the idea. The said studies had seemingly become redundant over several decades, yet learning and doing of problem solving is still a problem. Furthermore, review of literature revealed that researches on metacognition and problem solving have also flourished. The existing attempts to synthesize metacognition constructs are still dated 1976 & 1979 of Flavell's metacognitive knowledge & regulation, Sternberg in 1980 & Brown in 1987 for their metacognitive regulations in problem solving related activities, Jacobs & Paris' declarative, procedural & conditional metacognitive knowledge; Zimmerman in 2002 & Pintrich in 2004 for their self-regulated learning; and Polya in 1945, Schoenfeld in 1981, and Garofalo & Lester in 1985 for their metacognitive regulation in MPS. Importance of beliefs in MPS also has several studies (Young, 2010). Studies in metacognition for the past seven years after the presented synthesized work of the pioneers were followed by others showing the use of these constructs such as developing high profile psychometrics testing instruments, and several attempts of mixed methods studies to explore the qualitative dimension. However, further elaborations on what's beneath or beyond these psychological constructs silently working behind MPS domain are still a continued quest.

Most recently, Sternberg & Sternberg (2017) emphasized that there is always a sort of "obstacle" that prevents one to successfully solve a problem. The researcher intuitively agreed that there is something beyond this kind of mental activity requiring a much larger understanding of how bits of mental skills and strategies necessary for MPS should be put together for it to work. Taking a subjective-constructivist point of view with combined rationalist and empiricist philosophical standpoint tells that the identification of specific metacognition as the higher executive processes involved in MPS may serve as the new metacognitive knowledge domain vital to the learning of problem solving in a mathematical context.

In response to the aforementioned learning need, this study generally aimed to determine possible presence and interrelationships of these metacognitive processes as metacognition in successful MPS. Specifically, it sought to 1) describe the qualities of the metacognitive solutions that led to the right solution; 2) identify factors that contributed to the emergence of these qualities; and 3) determine the stages of problem solving that required the said metacognitive processes.

2. Method

Guided by a post-positivistic perspective under a qualitative research method, the constructivist researcher found the covert metacognitive phenomenon from 11 key informants. Combined criterion, homogenous, and theoretical sampling were realized to determine 15-17 years of age learners and identified to have above-average cognitive ability in OLSAT*8, with distinguished mathematical ability in their mathematics class and competition performances.

Strauss & Corbin (1998) used the grounded theory approach was utilized to guide and attain the purpose of this study. Data sources include self-report and interviews, written problem solutions, actual and video-records observation. Experts' participation from the fields of mathematics, psychology, and education ensured the validity and reliability of this study's instruments and methodology. A pilot administration of data collection was conducted in a psychology laboratory. Panel of experts observed, evaluated, and gave feedback on the quality of the said process for improvement in the actual implementation of the procedure. MPS competencies and test items covered were comprehensively studied, prepared, and tried out. Utilization of interview and observation guide was also tested for its feasibility. The Otis-Lennon Scholastic Aptitude Test version 8 (OLSAT*8) was chosen and conducted to determine key informants' cognitive ability upon the recommendation of a psychometrician.

Given the participant-observer constructivist role of the researcher, a rigorous process of emphatic introspection led into the development of cross-matching analysis process using three sources of data. This was realized to be a significant need in establishing metacognitive phenomenological open and axial codes of metacognitive processes and its qualities, consequently helpful in the selective coding phase of data analysis. Transcriber, inter-coders, and debriefers assisted in the process of analyzing data and interpreting formulated codes. This researcher-developed emphatic introspection and cross-matching analysis processes were conceptualized to ensure validity and reliability of the codes of metacognitive processes and its corresponding qualities. Interrelationships among these codes were derived from the determination of the nurture or nature factors that contributed to the emergence of the said metacognitive processes instrumental in successful understanding, conceptualization, and execution of the problem's solution. In the final analysis stage, selective coding formulated the emerging grounded theoretical framework of metacognition in successful MPS.

Ethics requirements were addressed. The researcher ensured the careful establishing of mutual understanding and trust-building among schools, participants, and families of the participants. Ethical concerns were considered conscientiously in ensuring that the process mentioned above was observed; and, in carrying out all the necessary procedures of the study-- especially in those parts where participants were highly vulnerable-careful treatment of the data was done. The selection of research design, context, and the procedure discussed above was carefully considered to ensure minimum ethical concerns. Hence, conscientious selection and participation of key informants were given much attention. Schools' approval was sought with a formal letter of intent sent to the office of the school heads for each of the identified schools. Ethical considerations of this study were properly guided by the ethical standards prescribed by the Board of the Social Research Ethics Review Committee (SRERC) of the researcher's institution. Important benefits were also explained to the parents, and matter of confidentiality was given assurance to be kept. FPIC forms were given only to students who have committed themselves to voluntary participation and have signed the given assent form. Photos, videos, and other documents containing direct information about these key informants' identities were kept confidential during the conduct of the data gathering process until the writing of this research paper.

3. Results and Discussion

Open codes revealed seven metacognitive processes themes, namely: 1) metacognitive knowledge of the typology of mathematical problems; 2) metacognitive knowledge of the nature of mathematical problems; 3) metacognitive awareness of mathematical knowledge and thinking; 4) metacognitive knowledge of personal strengths; 5) metacognitive knowledge of problem solving emotions and attitude; 6) metacognitive knowledge of thinking associated with bodily motion experiences; and 7) metacognitive solution qualities.

In this study, the first three of the six metacognitive processes are identified and clustered as the problem solving task-related metacognition. On the other hand, the next three are grouped together as the metacognitive affect and motion experiences. The seventh is termed as the metacognitive solution control.

Metacognitive knowledge of mathematical problems typology. This refers to the reflective thinking of key informants' own awareness of the type of problem they were presented with, how familiar a particular math problem is for them, or, if it was already encountered or taught in the classroom.

One key informant claimed, "It's routine 'coz I p-ph… encountered in class… about interest (garble)." And, the other one said, "It's non-routine because, it's not... discussed in the classroom. It's something that it's applicable to daily life (pause)…there is no fixed solution for the problem."

These findings conform with Garelick's (2013) idea stating that routine problems are those which are algorithmic by nature, have prescribed methods, and with predictable approaches; unlike non-routine problems that are considered to be more challenging, they may have no direct relevant applications or contain identified transferrable mathematical skills to deliberately develop.

Metacognitive knowledge of the nature of the problem. Open codes reveal that this metacognitive process indicates key informants' meta-awareness of a familiar math problem that is similar with other problems encountered.

As expressed in a key informant's narratives, "It's… like a combination again…! Like in number 8 (referring to a test item)… it's a combination … combination of… 2 persons, umm, working together…"

One's thinking about a familiar mathematical problem exhibited key informants' meta-awareness of certain mathematical knowledge related to the said problem. Usually, it involves association with relevant or similar mathematical knowledge structure learned in problem solving lessons. Interview codes revealed that familiar problems were reflectively believed to be easier to understand in comparison to those that were newly encountered.

A key informant affirmed, "… I was able to read this kind of problem in the first test, so I kind of had a grasp of what it really meant. So, I was able to understand this question. I think it… the question is quite easy. Easy for me so I was able to do it quickly."

The narrative shows that this metacognitive process on familiarity helped in the process of understanding a given math problem. Hence, it indicates that this metacognition provides support in the conceptualization of the kind of mathematical solution strategy that led key informants to answer either routine or non-routine problems correctly. Metacognitive familiarity to a given problem, however, does not necessarily guarantee the ease of the problem solving process they experienced. But the easiest problems are found to be in a set of routine problems rather than the non-routine ones. It shows that non-routine problem solutions indicate a metacognitive process that exhibits key informants' meta-recognition of familiar math problems experienced to be either easy or difficult than the routine ones due to clarity or ambiguity of understanding, respectively. The performance of the key informants in each problem solving item even revealed more successful routine solutions.

Metacognitive awareness of mathematical knowledge and thinking. This illustrates the meta-awareness of the key informants about the mathematical knowledge and thinking patterns and strategies they do in the process of solving either type of mathematical problems. Open codes specifically revealed that meta-awareness of mathematical knowledge covers key informants' meta-recognition of mathematical concepts and procedures involved in the right problem solutions they made. On the other hand, the metacognitive knowledge of one's own thinking used in the process of MPS exhibits several reflective thinking patterns (e.g. analysis, understanding, etc.) that utilize the said mathematical knowledge. The given narratives below explicitly provide evidence for this observation, as stated:

Key Informant 1: "I think it's all just in my stocked knowledge… about, umm, reduction of percentages. Everything from the adding up to operations… subtracting, multiplying, dividing and the… to exponents and …uhm…qualities/inequalities… uhm… also many properties like addition properties…"

Key Informant 2: "Umm. It is the way to solve the problem. The equation needed- fractions."

Key Informant 3: "Ahh, I first analyze the given and try to understand the question."

Metacognitive knowledge of personal strength in MPS. This theme of metacognitive process expresses the meta-awareness of key informants about their personal strengths in MPS in terms of three emerging sets of metacognitive idea, namely in 1) mathematical knowledge and learning experience; 2) mathematical solution strategies used; and (3) time consciousness and speed. These metacognitive processes observation were obtained from the interview and written solution open codes of routine and non-routine problems. Some of the key informants' narratives below indicate this observation:

Key Informant 1:"Uhmm… I think I am… I know the basic concepts well. And then, with the right Geometry – umm, rather… calculations on… like, when you memorize the values…it must be precise. Then everything will follow."

Key Informant 2: "I know how to solve them even though without a formula."

Key Informant 3: "So, umm. I think my strength in my... (Garble)... In the problem is that I read it to end... So that when I finished reading the problem I will not be able to forget the things that came into my mind."

The first two metacognitive personal strengths are commonly observed in routine problems. On the other hand, metacognitive awareness of time and speed consciousness is a dominant strength found in non-routine problems together with mathematical thinking. Time consciousness is considered as one of the metacognitive qualities or control discussed in the next section.

Metacognitive knowledge of emotions on beliefs, values & needs, or attitude in MPS. This theme in metacognitive affect processes exhibits key informants' meta-awareness of the set of emotions or attitude they experienced while solving a mathematical problem. Interview codes of both types of problem solutions revealed that this covers one's belief, needs, or values about certain mathematical conditions necessary to achieve successful MPS solutions.

One of the key informants expressed, "Umm…I understood this problem and know that I would be able to get the correct answer, unlike other problem which I have few ideas on how to solve it. I have more ideas in this problem than the others." On the other hand, another one demonstrated how feelings are intertwined to the value of thinking: "Umm…this was something new and it was... it required some critical thinking. I really need to think..."

Emotion is defined in psychology as a complex state of feeling that results in physical and psychological changes that influence thought and behavior. In a bigger sense, attitude encompasses this set of emotions, beliefs, and behaviors toward a particular object, person, thing, or event (Cherry, 2017). These two ideas are considered to be components of the affective domain.

This reflective state of feeling expresses a metacognitive emotion that is vital in developing a good attitude toward MPS. The metacognitive affect also validated another metacognitive sense of satisfaction when a problem is solved quickly. The metacognitive knowledge of the value given to speed with implied value of time mattered much. It was repeatedly emphasized in interviews about dimension and investment problems, "I'm happy… satisfied because it's very fast and correct!." The value of saving time and accuracy for them is very important. Similar to non-routine metacognitive affect, the consciousness of time, speed, and accuracy had developed that sense of good attitude that implicitly led in successfully answering routine math problems.

Metacognitive knowledge of mathematical thinking associated with hand and eye motion experiences. This theme of metacognitive process demonstrates a state of meta-awareness of being engaged in one's own thinking while bodily movements appeared to be incognito as coded and validated from key informants' interview responses, actual observation, and video-records reviews.

This study of investigating metacognitive processes in motion involved in MPS observed findings with resemblance to the 'embodiment theory' as social and psychological scientists call it. Embodied cognition theorized that mind works affect the motor system of the body, and in return, the body movements signal a new idea to the mind (Sternberg, 2017). However, Wilson & Golonka (2013) explain that embodied cognition should be understood differently. The brain is not the only source of learning or deliberate chain of command, but also the "rapid reflexes of body motor systems that provide shortcuts to very long complex mental processes". Problem stimulus that requires movements is naturally dealt with faster than going through heavy mental processing.

A key informant described explained own demonstrated action: "…Look upward! I try to remember if what formulas are needed for the problem…(Garble) I tried to analyze if I can see a pattern…then I read again…I tried to write away what I have thought…then if I can –can't see any, I look up again…thinking again." Another said, "Umm…I'm resting on my chin…on hand, analyzing the problem being read if what is asked and given…"

The open codes in this study saw this observation as an interplay of the mind and the motor, popularly known as the 'psychomotor'. This is similar to the so-called psychomotor intelligence or ability, where both body and mind meet at an unprecedented and inexplicable manner for execution (e.g., sports or bodily competition events champions). Not one between the two is the master, but they are in an equilibrium state. This can be associated to the study of conative domain that embodies all-the indivisible interconnectedness of the cognition, affect, and action. There has to be a sense of balance among these three to possibly make desirable action happens. However, it is not the usual case in which all that is being thought about have corresponding explicit actions or bodily motions all the time. Only those thoughts that involved deep and complex processes like MPS can manifest unnoticeable fine or gross motor motion.

The Metacognitive Solution Qualities (MSQ). This refers to the seventh theme of metacognitive processes. As shown in table 1, open codes from written solutions and self-report or interview responses discovered these metacognitive MPS solution qualities as the emerging ten micro-metacognitive processes. These qualities include diagnostic, scanning, searching & referencing, direction steering, rectification, restoration, pattern-seeking, predicting, natural flipping of thought, evaluating and controlling time. They were observed to be regulating the six other metacognitive knowledge relating to MPS task and the metacognitive affect and motion experiences mentioned above that are responsible for the achievement of successful routine or non-routine mathematical solution strategy. Hence, they are also termed in this study as the metacognitive solution controls.

Table 1 provides a general description of each quality and sample narratives culled from the open codes of interview and written significant responses of the key informants' problem solutions.

Table 1. 10 Themes of Metacognitive Solution Qualities of Successful MPS

Metacognitive Solution Qualities Themes	Routine Problems Open Coding Narratives	Non-Routine Problems Open Coding Narratives
1. **Diagnostic** is a micro-metacognitive solution quality that detects weakness, difficulty, or strengths.	"... And I arrived at this equation. Then, at first I was not sure because my answer was 50 hours which does not make sense!"	"I just had difficulty in understanding the question which is the highest percentage or ratio of minority to majority paint color..."
2. **Scanning, Searching, & Referencing** is a micro-metacognitive solution quality that looks, finds, or uses existing idea.	"Because, many times before, I would always encounter this problem and I do not know how to solve it. So, after, like 5 times of failing, I try to search for it. And then, remb... remember it... On how to solve it."	"By thinking all possible outcomes of the process (garble), it could reduce your errors... knowing the outcome you could know the possible ways or processes that you could use to tackle the problem."
3. **Direction Steering** is a micro-metacognitive solution quality that creates direction towards its goal.	"Then I tried, ahh, increasing the other value with higher interest so that I will get an increase in the possible interest."	"The other one will be, in the way, the imbalance and it would be saying that the other one is different and the other one is not. So, you just have to confirm another more... Than other apple to know this of the 3 is different."
4. **Rectification Effect** is a micro-metacognitive solution quality that corrects the detected error or mistakes.	"I arrived at this equation. Then, at first I was not sure because my answer was 50 hours which does not make sense because it would take too long... 50 hours to travel 50 miles...that's 1 mile per hour!"	"Because of the adding of the 1/8. I thought it would be 9/9 but it was wrong since the 8/8 plus 1/8 is supposed to be 9/8."
5. **Restoration** is a micro-metacognitive solution quality that automatically retrieves alternative mathematical strategy that was previously thought.	"Then when I was unable to do it I went back to my factoring... then got the right answer!"	"...I already tried fractions because I thought it was easy here. But much more complicated when I got here so, I tried to went back to percentage.
6. **Pattern Seeing or Seeking** is a micro-metacognitive solution quality that sees patterns of thought over related problems of similar solutions' structure, or finds patterns through series of numerical trials.	"I tried to find another solution. This is the first one (pointing to his answer)... this is the second... still the same. They are exactly the same."	"If I already notice, umm, a pattern, I think it would... already... I could already start progressing to the answer already..."
7. **Predictive** is a micro-metacognitive solution quality that foresees certainty of success due to advantage of the conceptualized mathematical strategy.	I understood this problem and know that I would be able to get the correct answer, unlike other problem which I have few ideas about on how to solve it. I have more ideas in this problem than the others."	"When you balance the 2 groups and they are the same it means there's no different one...They are all the same."
8. **Evaluative** is a micro-metacognitive solution quality that ensures quality and certainty of the whole conceptualization and execution process of problem's mathematical strategy and outcome.	"I am thoroughly reading my solution and my process if there's something... wrong... in the process...I try to check if there's something wrong, or missed... missed."	"So, I tried to add it and it's more than 1 gallon. And, this one has missing some parts so, it's wrong.
9. **Natural Flip of Thought** is the natural ability of thinking to execute sudden total understanding after sensible outcome is derived.	None	"...I already understood because I was able to answer the question."
10. **Time Consciousness & Control** is a micro-metacognitive solution quality that regulates time during the process of conceptualizing a mathematical strategy.	None	"I already have stopped because I still have problems to deal into... (referring to other test items)"

Diagnostic. Open coding analysis identified a metacognitive solution quality (MSQ) that is diagnostic by nature as expressed, for instance, by a key informant: "I just had difficulty in understanding the question which is the highest percentage or ratio of minority to majority paint color..." This demonstrates the quality of one's thinking to detect its weakness or difficulty, or strengths along the experiences in problem solving processes such as understanding a mathematical problem's scenario or context and conditions, or main idea; formulation of a solution; the execution of mathematical procedure or computation in a problem solving task. It is noticed to be a kind of natural self-diagnosing tendency of the mind to distinguish its own capacity of processing these thought processes when finding solutions in both types of mathematical problems.

Scanning, Searching, and Referencing (SSR). This cluster of MSQs contains patterns of thinking processes observed to be facilitating thoughts during the understanding of the problem's scenario, conditions, or main idea that led to the reflective process of conceptualizing a mathematical solution strategy and correct solution outcome. Scanning is identified as a quick recall of several relevant mathematical concepts or procedures to use. Searching is a metacognitive quality of thought that looks for relevant mathematical knowledge or solution strategy. Referencing is a natural tendency to make use of previous problem solving experiences, relevant mathematical knowledge and thinking for a familiar problem or similar mathematical strategy applicable to a given problem. These MSQs were observed to make sense and figure out a mathematical solution strategy for a given problem scenario.

Direction Steering. This refers to MSQ that creates regulated direction toward its problem solving goal- from making of assumptions to formulating of equations, from trying out different forms of mathematical strategy to procedural execution, from calculating numerical implications to ascertaining outcome's accuracy. This thinking behavior of thoughts was observed to be common in both types of mathematical problems.

Rectification Effect. Open codes recognize this MSQ as a natural tendency of the thought to correct the detected error or mistakes about a mathematical knowledge and thinking, mathematical solution strategy, or procedural execution involved in MPS.

Specifically, this reveals the ability of the thought to reconstruct translation or representation of the problem's given condition or the main problem idea when an error is identified. The ability of the thought to correct its thinking generally covers its tendency to detect inconsistencies in mathematical concepts, mathematical procedures or solution's general process. This allows for the thought to find that specific flaw in order to remediate. In other words, rectification is an observed consequential action or effect of the diagnostic tendency of the thought in identifying weaknesses or difficulties experienced during the formulation of solution in both types of mathematical problems.

Restoration Effect. This exhibits a character quality of the mind that automatically retrieves previously thought solution paths when stumbling blocks are diagnosed. This observation is similar to Palmeri's idea of "automatic processes versus controlled processes". As cited by Sternberg, Palmeri (2003), automatic processes may involve no conscious control. They demand a very small amount or no effort at all. Multiple or simultaneous processes may occur at once or, at the least, quickly with no particular sequence. Thus, they are also called "parallel processes".

Open codes from written responses further found this metacognitive process in some direction steering thoughts. The direction steering quality has allowed one thought to shift from one point to another for several paths with related experience of thought processes' diagnosis, SSR and possible rectification until it finally attained the correct solution outcome. Restoration effect happens when the path automatically returns to where the first thought process has started at certain point.

Pattern Seeking or Seeing. They are traces of metacognition tendency to see or find patterns of thought over related problems or solutions processes structure of the past experiences or patterns of reasoning that is dominantly needed in the present condition. This metacognitive thinking over what is posed as a problematic condition also follows certain metacognitive processes pattern that works from simple referencing of a set of similar problems' scenario and solution processes to a more complex, intuitive, or insightful decoding of key techniques in a chosen path of numerical random trials. There are key informants whose strengths in patterns seeing or seeking are in numerical trials and reasoning, but there are some who find patterns in schematic or structural identities of mathematical problems.

Predictive. This MSQ is observed in this study as the underlying reason of one's certainty when the solution path is leading towards the right direction. Key informants foresaw an advantage of the assumed solution process or procedure in routine or non-routine problems. Procedural assumptions of predicted advantage were strongly justifiable accompanied by accurate understanding or visualization, intuitive or insightful cues, and exhaustive conceptual induction or deduction of the whole solution path.

Evaluative. This MSQ is bigger than 'diagnostic' ability in a sense. The latter detects specific or singular weakness or strength along the process of exploring and monitoring the formulation of a mathematical thinking strategy or solution path. However, evaluative ability encompasses a system of solutions analysis- mathematical solution strategies are being conceptualized, or where mathematical procedures or computations are about to or being executed. It also includes decision-making or picking choices from mathematical strategies or solutions, at hand, based on one's mastery of skills for usage. Check-ups for mathematical procedural reasonability, consistency, and outcomes accuracy are also part of this ability.

Natural Flipping of Thought. This study has also observed a unique and interesting instance where a full understanding of the problem scenario is automatically thought only after arriving at a suspected valid or reasonable answer. This quick or automatic event of thinking is termed by the researcher as the "natural flip of thought". Open codes show that a flip of thought accompanying a suspected correct answer was observed to be a result of some trial and error procedures or random symbolic computations- the non-verbal way. It is likened to the pieces of puzzle that have to be figured out before meaning is appreciated.

Time Consciousness & Control. This quality refers to one's metacognitive thinking regulation of responses to a math problem due to a time demand. Key informants were observed to be particular with the time especially when they are aware of the complexity or difficulty of the problem, and the numbers of the problem they have to go through even if no time limit was stated. This observation is common when key informants are solving mathematical problems that are claimed to be non-routine.

The Nature and Nurture Factors that Influence Metacognition Solution Qualities (MSQ)

The natural factors that contributed to the emergence of some of these MSQs are found in this qualitative study as the inherent or natural qualities of thinking patterns or other external reasons that cannot be attributed directly to school learning or training.

These factors were observed to include natural verbal (e.g. language factor) or non-verbal numerical ability, intuition, insights, emotions and mental attitude, and some of these micro-metacognitive processes or qualities are responsible for the occurrences or regulation of others. These factors for such metacognition qualities to surface can be attributed also to the very high cognitive abilities of individuals who served as the primary source of this highly cognitive study. OLSAT*8 measured both their verbal and non-verbal ability which initially determined their logical reasoning and abstract thinking. Generally, these key informants possess above average performance range from 81 to 99 percentile rank of the norm.

Traces of non-verbal ability are found in some of the written responses of the key informant. Computational trials are used as a way to arrive at a correct answer even with lack of understanding. It simply indicates the natural occurring tendency to preferably rely on one's mathematical-logical ability more than the fluid reasoning of the verbal ones.

On the other hand, factors brought by 'nurture' refer to those that exhibited explicit influence of learned or acquired mathematical knowledge and thinking skills. These factors came out to include awareness of mathematical problem written structure, its typology and nature, awareness of strength or weakness in mathematical knowledge and thinking skills and strategies acquired, and the classroom learning and home training experienced.

In general, these factors, either nature or nurture, influenced the metacognition solution qualities observed to be important in successful MPS. Examining back the responses of the key informants found in the interviews- written problem solutions and observation- it occurred to the researcher that their personal strengths are pointed to the metacognitive knowledge that they possess about mathematical content and the related cognitive skills which worked when confronted with a problem solving task. However, these responses do not directly state whether nature or nurture causes the ability to successfully solve a given mathematical problem. But the complementary function of the two can be assumed.

The Emerging Metacognitive Stages of a Successful Mathematical Problem Solving

The self-report or interview responses, written solutions, and observation open codes present the observed stages in the actual processes involved in MPS. These emerged as: a) the big understanding process; b) the conceptualization of a mathematical solution strategy; and c) the execution of procedural or computational processes governing the said problems' solution. These stages were observed to dominantly involve metacognitive mathematical thinking patterns and varied solution strategies. They are influenced by the regulated metacognitive knowledge of the typology and nature of the problem being solved, the metacognitive emotions or attitude and the prevailing hand and eye movements in metacognitive thinking.

Big Understanding Process Stage. This stage manifests a set of metacognitive processes which exhibits key informants' utilization of the seven metacognitive processes involving the regulation of the problem solving task-related knowledge, thinking, emotions or attitude and some bodily gestures in order to mathematically represent routine or non-routine problem's scenario and provide accurate solution outcome for it.

This stage also emerged to subsume all other metacognitive processes directly involved in the whole MPS process. It includes some metacognitive indicators of thinking such as: (1) verifying thoughts through writing, (2) analyzing the problem while reading, (3) recognizing difficulty of the problem due to mathematical knowledge involved and time implication, (4) visualizing while reading, (4) justifying visualized images, (5) recognizing similarity in problems, (6) recalling and searching previous successful solution done in similar problem, (7) making references from visuals of similar problems, (8) decoding basic mathematical idea and arithmetic concepts involved, (9) inferring and drawing conclusions about relationships of variables, (10) translating problem's given conditions into mathematical expressions or drawing representations of the unknown, (11) reasoning interchangeability of variables' placement in an equation, and (12) and making sense of the whole solution process, and many other more.

Table 2 shows the sample narratives of the open codes that reveal the metacognitive processes involved in the big understanding process theme.

Table 2. Big Understanding Processes in Mathematical Problem Solving Sample Narratives

Themes of Metacognitive Stages in Mathematical Problem Solving	Metacognitive Stages in Routine Problem Solving Sample Narratives (SN)	Metacognitive Stages in Non-Routine Problem Solving Sample Narratives (SN)
Theme 1: Metacognitive Understanding Process	dominantly brought by some metacognitive thinking patterns associated with personal strengths and value of basic math concepts, reading and visualizing problem scenario or main idea SN: "I only pictured it after I got the... correct answer..." SN: "I read all. Uhm, I thought about how to solve it... and by using elimination...of each variable to get X and Y."	dominantly brought by some metacognitive thinking patterns that involved visualizing problem, trying out assumptions, maneuvering from false assumption, reconciling meaning, etc. SN: "Ahh, yes...the first statement I was sure that I was correct. I understood how he, the, he did it...how the transfers were made,"

Conceptualization of Mathematical Solution Strategy Stage. This stage shows the metacognitive regulation of key informants' big understanding process in order to formulate a mathematical solution strategy. Specifically, this stage has synthesized evidence that is pointing to a generic nature of 'planning' and 'understanding', an idea experienced in both routine and non-routine MPS.

The conditions on metacognitive solution qualities have hypothesized the possible danger that mental entrenchment, or the term "planning", is posing to problem solving learners. This may convey a false message stating that accomplishing MPS cannot happen unless there is planning. As indicated in the experiences of some key informants, the concept of planning was viewed differently. They understood it as a sort of complex or scientific and more systematic preparation before it has to be carried out.

Likewise, this study also realized that 'understanding' could have sent a wrong signal that a problem may never be solved unless everything is fully understood first.

Hence, qualitative findings discerned understanding as a continuous process while conceptualizing the whole mathematical solution strategy, encompassing both basic understanding and forethought planning, with all relevant mathematical knowledge and thinking governing the given mathematical problem structure at work. In other words, more time is being used in conceptualizing the mathematical solution strategy than the computational or algebraic operations execution part.

Table 3 shows the examples of the narratives in open codes that made up the theme of the metacognitive conceptualization of mathematical solution strategy.

Table 3. Conceptualization of Mathematical Solution Strategy Sample Narratives

Themes of Metacognitive Stages in Mathematical Problem Solving	Metacognitive Stages in Routine Problem Solving Sample Narratives (SN)	Metacognitive Stages in Non-Routine Problem Solving Sample Narratives (SN)
Theme 2: Metacognitive Conceptualization of Mathematical Solution Strategy	• focused on routine metacognitive mathematical knowledge and thinking patterns and strategies influenced much by metacognitive typology and nature of problems previously experienced, which appeared to involve formulating equations, or trial and error approaches in conceptualizing routine mathematical problem solutions. SN: "Umm. It is the way to solve the problem. The equation needed- fractions." SN: "Ahh, I first analyze the given and try to understand the question." SN: "I think when it comes to problems, uhm, that... uhm, I forget about the formula... sometimes, I resort to trial-and-error because most of the time, it works. It only takes a long time but it still works."	• demanded by the big metacognitive understanding process encompassing the conceptualization and execution of the whole mathematical problem solutions, which brought heuristic mathematical thinking strategies like case analysis, inductive and deductive reasoning, insightful thinking, listing, drawing, guess and check, and making table. SN: ": ... Knowing the outcome you could know the possible ways or processes that you could use to tackle the problem. I just try to read and try to understand." SN: "First is, as it is in my paper that I just first use fractions but, it was kind of hard because...I, ang...I don't really quite understand the problem yet, so, I tried ahhh, to use percentages so that maybe I can try to...uhm..."

Execution of the Conceptualized Mathematical Solution Strategy. Self-report or interview and written open codes also found the seven metacognitive processes being regulated in executing a successful mathematical solution strategy in either routine or non-routine math problem.

The execution of mathematical solution strategy in routine problems commonly came out as the regulated application of algebraic rules in solving the formulated equations or basic numerical computations to follow the logic of basic numerical reasoning in seeking patterns in a trial and error strategy. The regulation of these processes ascertains the accuracy of the final answer. Similarly, the execution of mathematical solution strategy in non-routine problems is commonly observed in calculations that involved numerical equations or fluency of reasoning.

Sample narratives of the open codes comprising this theme of the mathematical solution strategy execution stage are shown in table 4 below.

Table 2. Big Understanding Processes in Mathematical Problem Solving Sample Narratives

Themes of Metacognitive Stages in Mathematical Problem Solving	Metacognitive Stages in Routine Problem Solving Sample Narratives (SN)	Metacognitive Stages in Non-Routine Problem Solving Sample Narratives (SN)
Theme 3: Metacognitive Execution of the Conceptualized Mathematical Solution Strategy	□ came out as metacognitive application of algebraic rules in solving the formulated equations or, basic numerical computations to follow the logic of basic numerical reasoning in seeking for patterns in a trial and error strategy SN: "I just lay-out the given... Old impri... older press is 2x the newer press. It is the sum of the reciprocals... Is equal to the total time... Reciprocal...." SN: "I add zeroes first so that it will-after the 8 so that it will become 2 digit by 2 digit..." SN: "Uhm...maybe multiplying decimals with whole numbers...Ah! N'this part I got... Umm, I paused for a little bit..."	□ observed as metacognitive calculations that involved numerical equations or fluency of reasoning SN: "I think when I... when I arrived at the answer I tried to trace it back to this part (pointing to his answer)... I think I tried to reverse everything... and If I get what the initial... initial given "r" then my... my answer is correct. SN: Yes... I think that... when it's here... I... when I got this 2/9 fraction... since it's total of 18... I was able to get 4. That's what's... then it was made clear that it would be... when I checked it 2/9 of this 4kgs. It could be 18.

To synthesize the emerging stages, it can be intuitively understood that the whole process of a successful MPS is a big metacognition phenomenon working under the seven metacognitive processes. As a whole, figure 1 presents a schematic diagram of the interrelationships among the emerging metacognitive processes [in green and dark and light blue coded boxes] and these metacognitive stages of MPS processes [in red coded box] that require metacognition to attain the right solution outcome.

Open codes have led this study to identify the mentioned three themes of metacognitive stages that are vital in a successful MPS. They also emerged from the analysis of the key informants' metacognitive solution qualities noted to be the control factor that is responsible for regulating their metacognitive task-related knowledge and metacognitive affect and motion experience while solving mathematical problems. As revealed in the discussion of the factors influencing each metacognition solution quality, the 'evaluative control' found to be regulating all components of the problem solving metacognitive processes and stages framework. This control factor is one of the ten micro-metacognitive processes that is responsible for ascertaining the soundness, reasonability, consistency, or advantage of the over-all 'mathematical solution strategy' or solution system's framework necessary to obtain an accurate MPS outcome.

Series of axial coding reveal the specific roles of each of these metacognitive processes that were clustered into three themes. These roles become the modifier of the six selective codes of Corbin & Strauss (1998), namely, 1) the causal condition that is influencing the core phenomenon, 2) the core phenomenon which is affecting the problem solving process, 3) the strategies which is the resulting effect of the core phenomenon, 4) the context which has a specific condition influencing the strategies, 5) the intervening variable which has general conditions affecting the strategies, and 6) the consequences which is the ultimate outcome of using the strategies.

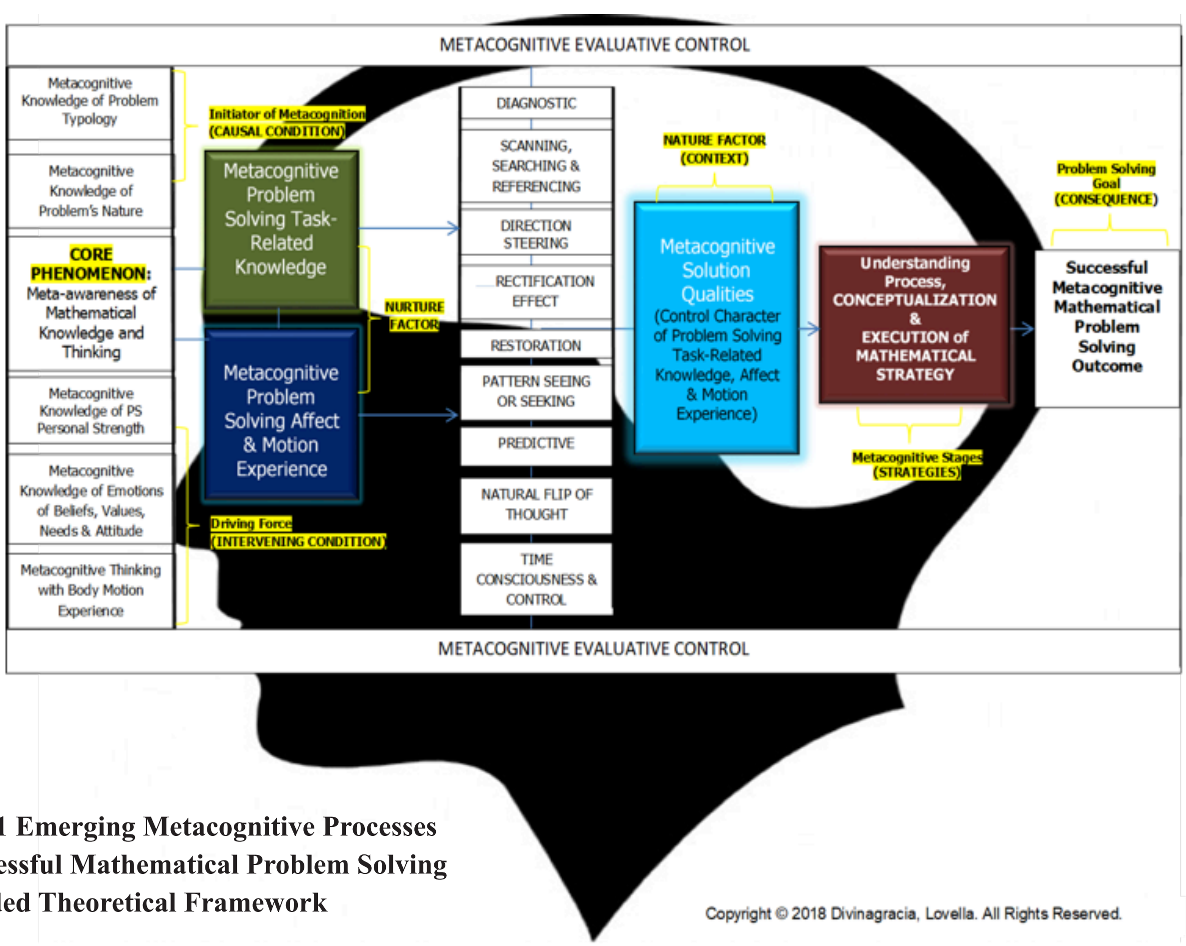

Figure 1 Emerging Metacognitive Processes in Successful Mathematical Problem Solving Grounded Theoretical Framework

Figure 1 further illustrates the selective coding [in yellow codes] of this study. The 'causal condition' was found to be the 'Initiator of Metacognition'. This covers the function of the first two metacognitive processes that made up the metacognitive problem solving task-related knowledge [green coded box]. They set the 'metacognitive knowledge and thinking' in motion, which actively mobilizes the big process of understanding, the conceptualization, and execution of a mathematical solution strategy. The active formulation of varied mathematical solution strategies has become the emerging 'strategies', which led to successful solution outcomes.

On the other hand, important intervening conditions affecting these mathematical solution strategies were found in the three metacognitive processes of the metacognitive affect and motion theme cluster [dark blue coded box]. They were observed to be the 'Driving Force' of the core phenomenon to continue its motion. Central to this mathematical problem solving process is the Core Phenomenon of 'Metacognitive Mathematical Knowledge and Thinking' mentioned.

As illustrated in the schematic framework, both the green and the dark blue coded metacognitive processes were found as essential nurturing factor (Nurture Factor) significantly affecting the context or specific conditions directly influencing the 'mathematical solution strategies'. These specific conditions are the micro-metacognitive processes which serve as the metacognitive solution qualities (MSQ) instrumental in the attainment of appropriate and accurate mathematical strategies and outcomes 'consequence'. This observed regulating MSQ, supported by the nurturing metacognitive processes, is considered in this study as the natural influencing factor (Nature Factor) responsible for the said MPS success.

Finally, as shown in figure 1, selective coding classified the over-all interrelationships found among these metacognitive processes as either a nature or nurture factor influencing the success of the whole metacognitive process of MPS. A black and white head brain clipart by Clipground (2017) was used as a background to emphasize the mental processes.

4. Implications and Recommendations

The set of above average cognitive abilities and the learning experiences of the key informants behind these processes served as the ground for having these descriptions of the metacognitive phenomenon. In addition, the routine and non-routine typology of the mathematical problems chosen, which is also limited to its competency domain of numerical reasoning and algebraic thinking should also be cautiously considered in order to understand the grounded theoretical framework of successful MPS. The prescribed covered content domain includes the mathematical knowledge on concepts and procedures involved in ratio and proportion, or percentage, probability, algebraic expressions, equations, and simultaneous equations or systems of equations, variations, inverse relations, fractions, decimal, and applications of basic arithmetic operations.

Bearing in mind the specific findings of this study and the emanated grounded theoretical framework of metacognitive processes in successful MPS, the researcher realized the importance of its implication to home and school education. The country's challenges confronting the teaching and learning of problem solving has already surpassed decades of studies and researches that are seeking for its improvement and yet these challenges continue to prevail despite curriculum reforms in the field of 21st century mathematics education on a global perspective. Based on the given findings, implications and recommendations therefore emphasized the need for teaching-learning integration of metacognitive thinking skills and strategies, metacognitive affect and bodily motion, micro and macro metacognitive experiences, curriculum review, and conduct of further studies and local researches in varied disciplines and possible wider educational perspectives.

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